

# Vibrating Superconducting Island in a Josephson Junction

J. Fransson,<sup>1,2,3,\*</sup> Jian-Xin Zhu,<sup>1</sup> and A. V. Balatsky<sup>1,4</sup>

<sup>1</sup>*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

<sup>2</sup>*Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

<sup>3</sup>*Department of Physics and Materials Science, Uppsala University, Box 530, SE-751 21 Uppsala, Sweden*

<sup>4</sup>*Center for Integrated Nanotechnology, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

(Dated: July 14, 2008)

We consider a combined nanomechanical-superconducting device that allows the Cooper pair tunneling to interfere with the mechanical motion of the middle superconducting island. Coupling of mechanical oscillations of a superconducting island between two superconducting leads to the electronic tunneling generates a supercurrent that is modulated by the oscillatory motion of the island. This coupling produces alternating finite and vanishing supercurrent as function of the superconducting phases. Current peaks are sensitive to the superconducting phase shifts relative to each other. The proposed device may be used to study the nanoelectromechanical coupling in case of superconducting electronics.

PACS numbers: 85.85.+j, 73.40.Gk, 82.25.Cp

Vibrational modes (vibrons) and spins possess dynamical degrees of freedom and they have a large impact on electron dynamics. Peaks and dips in the differential conductance of molecular electronics devices [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] may indicate strong effects from electron-vibron coupling. Steps in the differential conductance have been observed, both experimentally and theoretically, in STM based inelastic tunneling spectroscopy (IETS) around local vibrational mode on surfaces [15, 16]. Effects from local vibrational modes on the conductance in molecular quantum dots and single electron transistor have also been investigated [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. These studies may have implications on charge-based quantum information technology.

More recently the field has been moving towards incorporating superconducting electronics into nanoelectromechanical devices, e.g. see recent work [28, 29]. One example of superconducting electronics in combination with nanomechanical setup is a Cooper pair shuttle. The setup proposed originally [30] was aimed at a steady state description of the mechanical assisted Cooper pair tunneling. We extend the analysis of Cooper pair shuttle to consider possible resonances between mechanical motion and ac Josephson effect. We here address dynamical aspects of this device where we focus on time domain in the presence of ac signatures in the electric current.

In this Letter we study Josephson tunneling in a system with a mechanically moving superconducting island between two superconducting leads. Mechanical oscillations of the island couples to the electronic tunneling. This coupling gives rise to an oscillator modulation of the Josephson current, such that the Fourier spectrum in general exhibits current peaks at  $\omega_J \pm \tilde{\omega}_0$ ,  $\omega_J \pm 2\tilde{\omega}_0$ , and  $2\omega_J$ , where  $\omega_J = eV$  and  $\tilde{\omega}_0$  are the Josephson and renormalized oscillator frequencies, respectively. In addition, the zero bias supercurrent peaks at the frequencies

0,  $\pm\tilde{\omega}_0$ , and  $\pm 2\tilde{\omega}_0$ , however, the peaks at 0 and  $\pm 2\tilde{\omega}_0$ , or  $\pm\tilde{\omega}_0$  vanish for certain combinations of the superconducting phases in the three components of the system.

The physical system we consider consists of a superconducting island between two superconducting leads, where the island is exposed to mechanical vibrations modeled with Hook's law force constant  $k_c$ , see Fig. 1. A bias voltage is applied across the junction. The Hamiltonian for the system is given by

$$\mathcal{H} = \mathcal{H}_L + \mathcal{H}_I + \mathcal{H}_R + \mathcal{H}_T. \quad (1)$$

Here the leads ( $L, R$ ) and the island ( $I$ ) are described by the BCS Hamiltonians as  $\mathcal{H}_{L(I,R)} = \sum_{p(k,q)\sigma} \varepsilon_{p(k,q)} c_{p(k,q)\sigma}^\dagger c_{p(k,q)\sigma} + \sum_{p(k,q)} [\Delta_{L(I,R)} c_{p(k,q)\uparrow}^\dagger c_{-p(-k,-q)\downarrow}^\dagger + H.c.]$ . We denote the creation (annihilation) operators by  $c_{p(k,q)\sigma}^\dagger$  ( $c_{p(k,q)\sigma}$ ), where the subscript  $p(k,q)$  is momenta in the leads and island, whereas  $\sigma = \uparrow, \downarrow$  is a spin index. Finally,  $\varepsilon_{p(k,q)}$  and  $\Delta_{L(I,R)}$  are the single-electron energies and

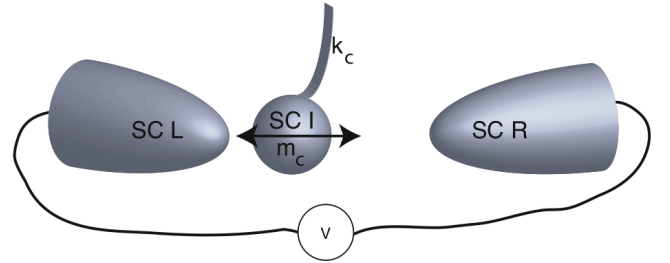


FIG. 1: Schematic view of the mechanically and electronically coupled superconducting island (SC I) to the superconducting leads (SC L and SC R). The cantilever superconducting island is modeled as a harmonic oscillator with spring constant  $k_c$  and mass  $m_c$ , placed between the infinitely massive superconducting leads. The device is biased with voltage  $V$ .

pair potential (gap function), respectively. Without loss of generality, we assume that the superconductors are of a conventional spin-singlet  $s$ -wave pairing symmetry, and consider the Josephson tunneling at zero temperature. The last assumption implies that the temperature is low compared to relevant superconducting gaps, e.g.  $T \ll \Delta_{L(I,R)}$ . To avoid thermal damping of an oscillator we also require low enough  $T \sim 10 - 100$  mK. The last term in Eq. (1) describes the tunneling between the leads and the island, i.e.  $\mathcal{H}_T = \sum_{pk\sigma} [T_{pk} c_{p\sigma}^\dagger c_{k\sigma} + H.c.] + \sum_{qk\sigma} [T_{qk} c_{q\sigma}^\dagger c_{k\sigma} + H.c.]$ , where the tunneling matrix elements  $T_{p(q)k}$  transfers electrons through insulating barriers between the leads and the island. The local vibrational mode of the island is in the linear coupling regime given by

$$T_{pk} = T_{pk}^{(0)}(1 + \alpha_L u), \quad T_{qk} = T_{qk}^{(0)}(1 + \alpha_R u), \quad (2)$$

where  $\alpha_{L(R)}$  describes the coupling between the tunneling electrons and the vibrational mode corresponding to the left (right) tunnel junction. The quantity  $u$  is the displacement operator for the oscillator. The tunneling matrix element  $T_{pk}$  is exponential in displacement  $u$ , thus, the assumed linear coupling is a good approximation for small  $u$ . This allows evaluation of  $\alpha_{L,R}$  in terms of the tunneling matrix elements and their distance dependence. We assume here a very general equilibrium geometry with no particular symmetry being required. The equilibrium point ( $u = 0$ ) of the mechanical oscillator placed within the junction could be placed anywhere in between the leads. The energy associated with the vibrational mode,  $\omega_0 = \sqrt{k_c/m_c} \sim 10^{-1} - 10^{-6}$  eV, is much smaller than the typical electronic energy on the order of 1 eV, the mechanical oscillations are very slow compared to the time scale of the electronic processes. This allows us to apply the Born-Oppenheimer approximation to treat the electronic degrees of freedom as if the local oscillator is static at every instantaneous location.

The current is derived using standard methods. For a given bias voltage  $eV = \mu_L - \mu_R$ , where  $\mu_\chi$ ,  $\chi = L, R$  is the chemical potential of the left ( $L$ ) and right ( $R$ ) lead, the Josephson current between the lead  $\chi$  and the island is given by [27] (setting  $\hbar = 1$ )

$$I_S^\chi(t) = J_S^\chi(\omega_\chi) [1 + \alpha_\chi u]^2 \sin(\omega_J^\chi t + \phi_\chi) - \Gamma_S^\chi(\omega_\chi) [1 + \alpha_\chi u] \alpha_\chi \dot{u} \cos(\omega_J^\chi t + \phi_\chi), \quad (3)$$

in applying the local approximation  $u(t') \simeq u(t) + (t' - t)\dot{u}(t)$ . Here we have introduced the phase difference  $\phi_\chi$  between the lead  $\chi$  and the island, and the Josephson frequency  $\omega_J^\chi = 2(\mu_\chi - \mu_I)$ . In Eq. (3),  $J_S^\chi$  is the amplitude of the Josephson current when the cantilever is

frozen ( $u = 0$ ), given by

$$J_S^\chi(\omega_\chi) = e \sum_{n \in \chi, k} \frac{|T_{nk}^{(0)}|^2 |\Delta_\chi \Delta_I|}{E_n E_k} \left( \frac{1}{\omega_\chi + E_n + E_k} - \frac{1}{\omega_\chi - E_n - E_k} \right), \quad (4)$$

where  $\Delta_\chi$  and  $\Delta_I$  is the superconducting gap in the lead  $\chi$  and island, respectively, whereas the quasi-particle energies  $E_n = \sqrt{(\varepsilon_n - \mu_\chi)^2 + |\Delta_\chi|^2}$  and  $E_k = \sqrt{(\varepsilon_k - \mu_I)^2 + |\Delta_I|^2}$ . The second contribution to the Josephson current in Eq. (3) has the amplitude

$$\Gamma_S^\chi(\omega_\chi) = e \sum_{nk} \frac{|T_{nk}^{(0)}|^2 |\Delta_\chi \Delta_I|}{E_n E_k} \left( \frac{1}{(\omega_\chi + E_n + E_k)^2} - \frac{1}{(\omega_\chi - E_n - E_k)^2} \right). \quad (5)$$

Using that  $I_S^R = -I_S^L$  for stationary bias voltages we write the total Josephson current  $I_S = I_S^L = (I_S^L - I_S^R)/2$ .

The Hamiltonian  $\mathcal{H}_J$  for the Josephson energy [27] is derived by requiring that the derivative of  $\mathcal{H}_J$  with respect to  $\phi_\chi$  yields the supercurrent given in Eq. (3). We find that

$$\mathcal{H}_J = \frac{1}{2} \sum_{\chi=L,R} \left( E_J^\chi (1 + \alpha_\chi u)^2 [1 - \cos(\omega_J^\chi t + \phi_\chi)] - \frac{1}{2e} \Gamma_S^\chi (1 + \alpha_\chi u) \alpha_\chi \dot{u} \sin(\omega_J^\chi t + \phi_\chi) \right) \quad (6)$$

where  $E_J^\chi = J_S^\chi/(2e)$ . This effective Hamiltonian captures the back action effect and reflects how the electronic degrees of freedom influence the mechanical oscillator. The classical motion of the shuttling island is described by the Hamiltonian  $\mathcal{H}_{osc} = p^2/(2m) + k_c u^2/2 + \mathcal{H}_J$ , which results in the classical equation of motion

$$m_c \ddot{u} + [\gamma_S(t) + \gamma_N] \dot{u} + k_c u = F(t). \quad (7)$$

Here, the driving force

$$F(t) = - \sum_{\chi=L,R} E_J^\chi \alpha_\chi (1 + \alpha_\chi u) \left[ 1 - \left( 1 + \frac{\omega_J^\chi \Gamma_S^\chi}{4e E_J^\chi} \right) \times \cos(\omega_J^\chi t + \phi_\chi) \right], \quad (8)$$

whereas the time-dependent damping factor  $\gamma_S(t) + \gamma_N$  contains the superconducting contribution

$$\gamma_S(t) = -\frac{1}{2e} \sum_{\chi=L,R} \Gamma_S^\chi \alpha_\chi^2 \sin(\omega_J^\chi t + \phi_\chi). \quad (9)$$

due to damping energy in and out of the superconducting carriers, and the external damping  $\gamma_N$ . We assume that

the damping into non-superconducting degrees of freedom is suppressed by the factor  $\exp(-\Delta_{L,R}/T)$ , as one would need to damp energy into normal excitations.

In order to make next steps analytically, we assume the system to be in the weak coupling limit. Moreover, since  $\Gamma_S^\chi \ll J_S^\chi$  the main physics is captured by neglecting the terms in the damping and driving force proportional to  $\alpha_\chi \Gamma_S^\chi$  and  $\alpha_\chi^2$ . The motion of the island is then given by

$$u(t) = u_0 \sin(\tilde{\omega}_0 t + \delta_0) e^{-\gamma_N t/2m_c} - \sum_\chi \frac{\alpha_\chi E_J^\chi}{k_c} [1 - \mathcal{A}_\chi \times \{\cos(\omega_J^\chi t + \phi_\chi) + \mathcal{B}_\chi \sin(\omega_J^\chi t + \phi_\chi)\}], \quad (10)$$

where

$$\mathcal{A}_\chi = \frac{1 - (\omega_J^\chi/\omega_0)^2}{[1 - (\omega_J^\chi/\omega_0)^2]^2 + (\gamma_N \omega_J^\chi/k_c)^2} \quad (11)$$

$$\mathcal{B}_\chi = \frac{\gamma_N \omega_J^\chi}{k_c} \frac{1}{1 - (\omega_J^\chi/\omega_0)^2}. \quad (12)$$

The first term describes the unperturbed motion of the island around its equilibrium position, where  $u_0$  and  $\delta_0$  are to be determined by the initial conditions, whereas  $\varepsilon_\pm = -\gamma_N/2m \pm i\tilde{\omega}_0$  with eigenfrequencies  $\tilde{\omega}_0 = \sqrt{\omega_0^2 - [\gamma_N/2m]^2}$  of the mechanical oscillations.

The other terms arise due to the coupling between the mechanical and electronic degrees of freedom. Using this expression for the mechanical motion of the island, we obtain the Josephson current approximately as

$$I_S^\chi(t) = J_S^\chi [1 + 2\tilde{\alpha}_\chi \sin(\tilde{\omega}_0 t + \delta_0) e^{-\gamma_N t/2m_c} + \tilde{\alpha}_\chi^2 \sin^2(\tilde{\omega}_0 t + \delta_0) e^{-\gamma_N t/m_c} - 2\tilde{\alpha}_\chi \sum_{\chi'=L,R} \frac{\tilde{\alpha}_{\chi'}}{K_{\chi'}} (1 - \mathcal{A}_{\chi'} \{\cos(\omega_J^{\chi'} t + \phi_{\chi'}) + \mathcal{B}_{\chi'} \sin(\omega_J^{\chi'} t + \phi_{\chi'})\})] \times \sin(\omega_J^\chi t + \phi_\chi), \quad (13)$$

where  $\tilde{\alpha}_\chi = u_0 \alpha_\chi$  and  $K_\chi = k_c u_0^2/E_\chi$ .

We now consider a few limiting cases in the symmetrically biased system, such that  $\omega_L = -\omega_R = \omega_J (= eV)$ , and we assume that  $T_{pn}^{(0)} = T_{qn}^{(0)}$  and  $|\Delta_L| = |\Delta_R|$ . Then,  $J_S^L = J_S^R = J_S$ , hence, we set  $E_J^\chi = E_J$  and  $K_\chi = K$ . Moreover,  $\mathcal{A}_L = \mathcal{A}_R = \mathcal{A}$ , and  $\mathcal{B}_L = -\mathcal{B}_R = \mathcal{B}$ , and we also assume that  $\alpha_R = -\alpha_L = -\alpha$ . Using Eq. (13) we find that the total current under those circumstances can be written as

$$\begin{aligned} \frac{2I_S(t)}{J_S} = & [1 + \tilde{\alpha}^2 \sin^2(\tilde{\omega}_0 t + \delta_0) e^{-\gamma_N t/m_c}] [\sin(\omega_J t + \phi_L) + \sin(\omega_J t - \phi_R)] + 2\tilde{\alpha} \sin(\tilde{\omega}_0 t + \delta_0) [\sin(\omega_J t + \phi_L) \\ & - \sin(\omega_J t - \phi_R)] e^{-\gamma_N t/2m_c} + \tilde{\alpha}^2 \mathcal{A} [\sin 2(\omega_J t + \phi_L) + \sin 2(\omega_J t - \phi_R) - 2 \sin(2\omega_J t + \phi_L - \phi_R) \\ & + \mathcal{B} \{2 - \cos 2(\omega_J t + \phi_L) - \cos 2(\omega_J t - \phi_R) + 2 \cos(2\omega_J t + \phi_L - \phi_R) - 2 \cos(\phi_L + \phi_R)\}] / K \end{aligned} \quad (14)$$

In the case where the phases  $\phi_\chi = 0$  the Josephson current thus reduces to

$$\frac{2I_S(t)}{J_S} = 2 \left( 1 + \tilde{\alpha}^2 \sin^2(\tilde{\omega}_0 t + \delta_0) e^{-\gamma_N t/m_c} \right) \sin \omega_J t. \quad (15)$$

In the undamped, or very weakly damped case, this expression predicts that there will be a dc component to the Josephson current whenever the bias voltage matches the Shapiro step value  $2\tilde{\omega}_0$ . On the other hand, there is no dc component to the current at the value  $\tilde{\omega}_0$  as is predicted for a single tunnel junction in Ref. [27]. The motion of the island between the two leads, clearly, has a compensating effect in the sense that the coupling of the mechanical oscillator and the tunneling electrons cancel their respective side peaks at  $\omega_J \pm \tilde{\omega}_0$ . These observations open up possibilities to excite cantilever motion by generating the Josephson current with appropriate frequencies. The features predicted from Eq. (14) at  $\phi_\chi = 0$ , that is,

the absence of Josephson current at  $\omega_J \pm \tilde{\omega}_0$  and  $2\tilde{\omega}_J$ , while there is a finite Josephson current at  $\omega_J \pm 2\tilde{\omega}_0$ , are clearly illustrated in Fig. 2. These plots show the Fourier transform of the Josephson current  $\log_{10} |2I_S(\omega)/J_S|^2$ , in the undamped case ( $\gamma_N = 0$ ), as function of the phase  $\phi_L$  and frequency  $\omega$ , for the fixed phase  $\phi_R = 0$  and two different frequencies  $\omega_0$ .

More interesting is the zero bias Josephson current found from Eq. (14). Assuming that the phases  $\phi_L = \phi_R = \phi$  we obtain

$$\frac{2I_S(t)}{J_S} = 4\tilde{\alpha} \sin(\tilde{\omega}_0 t + \delta_0) \sin \phi e^{-\gamma_N t/2m_c} \quad (16)$$

Hence, there is an ac component which is modulated by the eigenfrequency  $\tilde{\omega}_0$  of the moving island. The dashed lines in Fig. 3 signify equal phases  $\phi_\chi = \phi$ , at which only the ac component which is modulated by  $\tilde{\omega}_0$  is finite.

In the case of opposite phases,  $\phi_L = -\phi_R = \phi$ , we find

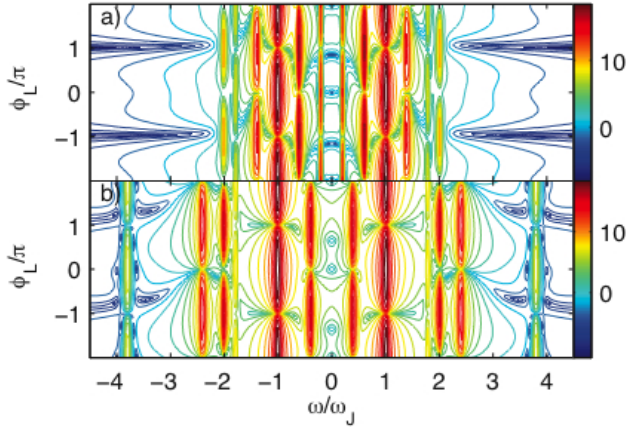


FIG. 2: (Color online) Fourier transform of the Josephson current  $\log_{10} |2I_S(\omega)/J_S|^2$  in the symmetrically biased and undamped system as a function of the frequency  $\omega$  and the phase  $\phi_L$ , for a fixed phase  $\phi_R = 0$ . Here we have taken  $\tilde{\alpha} = 0.1$ ,  $K = 0.6$ , and the oscillating frequencies a)  $\omega_0 = 0.4\omega_J$  and b)  $\omega_0 = 1.4\omega_J$ .

the Josephson current

$$\frac{2I_S(t)}{J_S} = 2 \left( 1 + \tilde{\alpha}^2 \sin^2(\tilde{\omega}_0 t + \delta_0) e^{-\gamma_N t/m_c} \right) \sin \phi \quad (17)$$

In contrast to the previous case, in this case there is a dc component depending on the phase  $\phi$ . Moreover, opposite phases give rise to an ac component which is modulated by twice the eigenfrequency ( $2\tilde{\omega}_0$ ) of the moving island. Opposite phases are signified by solid lines in Fig. 3, showing a finite dc component to the Josephson current, along with finite ac components at  $\pm 2\tilde{\omega}_0$ .

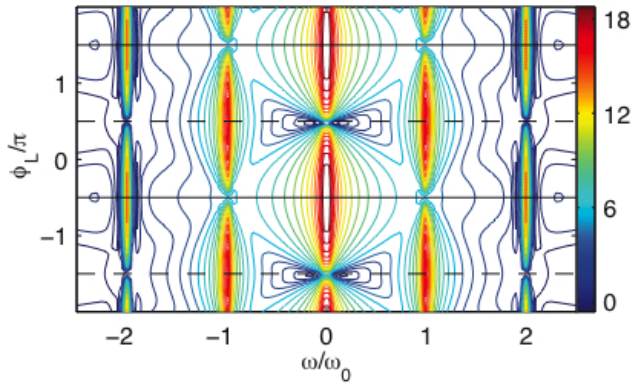


FIG. 3: (Color online) Zero bias Josephson current  $\log_{10} |2I_S(\omega)/J_S|^2$  in the symmetrically biased and undamped system as a function of the frequency  $\omega$  and the phase  $\phi_L$ , for a fixed phase  $\phi_R = \pi/2$ . Here we have taken  $\omega_0 = 1$ , while other parameters are as Fig. 2.

For an experiment of this type, taking  $|\Delta| \sim 10$  meV as relevant to MgB<sub>2</sub> [31] provides a sufficiently small mechanical damping. By acquiring a vibrational frequency of  $\omega_0/2\pi \sim 1$  GHz [32], it should be possible to tune the Josephson frequency such that  $\omega_0/\omega_J \gtrsim 0.1$ . A coupling strength  $\alpha/\omega_0 \sim 10^{-1} - 10^{-3}$  [33] would be sufficient to enable read-out of our predictions.

This work has been supported by US DOE, LDRD and BES, and was carried out under the auspices of the NNSA of the US DOE at LANL under Contract No. DE-AC52-06NA25396.

\* Electronic address: Jonas.Fransson@fysik.uu.se

- [1] R. P. Andres *et al.*, Science **272**, 1323 (1996).
- [2] M. A. Reed *et al.*, Science **278**, 252 (1997).
- [3] C. Kergueris *et al.*, Phys. Rev. B **59**, 12505 (1999).
- [4] S. Hong *et al.*, Superlattices Microstruct. **28**, 289 (2000).
- [5] J. J. W. M. Rosink *et al.*, Phys. Rev. B **62**, 10459 (2000).
- [6] J. Chen *et al.*, Appl. Phys. Lett. **77**, 1224 (2000).
- [7] D. Porath *et al.*, Nature **403**, 635 (2000).
- [8] R. H. M. Smit *et al.*, Nature **419**, 906 (2002).
- [9] J. Reichert, *et al.*, Phys. Rev. Lett. **88**, 176804 (2002).
- [10] A. Aviram and M. Ratner, in *Molecular Electronics: Science and Technology* (Annals of the New York Academy of Science, New York, 1998).
- [11] V. J. Langlais *et al.*, Phys. Rev. Lett. **83**, 2809 (1999).
- [12] H. Park *et al.*, Nature **407**, 57 (2000).
- [13] J. Park, Nature **417**, 722 (2002).
- [14] N. B. Zhitenev *et al.*, Phys. Rev. Lett. **88**, 226801 (2002).
- [15] B. C. Stipe *et al.*, Science **280**, 1732 (1998); Phys. Rev. Lett. **82**, 1724 (1999); N. Lorente *et al.*, *ibid.* **86**, 2593 (2001).
- [16] J. Fransson and A. V. Balatsky, Phys. Rev. B **75**, 195337 (2007).
- [17] N. S. Wingreen *et al.*, Phys. Rev. B **40**, 11834 (1989).
- [18] J. Kohler *et al.*, Nature **363**, 342 (1993); J. Wrachtrup *et al.*, *ibid.* **363**, 244 (1993); J. Wrachtrup *et al.*, Phys. Rev. Lett. **71**, 3565 (1993).
- [19] H. -A. Engel and D. Loss, Phys. Rev. Lett. **86**, 4648 (2001); Phys. Rev. B, **65**, 195321 (2002).
- [20] U. Lundin and R. H. McKenzie, Phys. Rev. B **66**, 075303 (2002).
- [21] J.-X. Zhu and A. V. Balatsky, Phys. Rev. B **67**, 165326 (2003).
- [22] K. Flensberg, Phys. Rev. B **68**, 205323 (2003).
- [23] A. Mitra *et al.*, Phys. Rev. B **69**, 245302 (2004).
- [24] N. M. Chtchelkatchev *et al.*, Phys. Rev. B **70**, 193305 (2004).
- [25] J.-X. Zhu *et al.*, Phys. Rev. Lett. **92**, 107001 (2004).
- [26] Z. Nussinov *et al.*, Phys. Rev. B **71**, 214520 (2005).
- [27] J.-X. Zhu *et al.*, Phys. Rev. B **73**, 064513 (2006).
- [28] K. Schwab, Nature **444**, 161 (2006).
- [29] M. Meschke *et al.*, Nature **444**, 187 (2006).
- [30] L. Y. Gorelik *et al.*, Nature **411**, 454 (2001).
- [31] B. C. Stipe *et al.*, Phys. Rev. Lett. **86**, 2874 (2001).
- [32] X. M. H. Huang *et al.*, Nature **421**, 496 (2003).
- [33] E. K. Irish *et al.*, Phys. Rev. B, **72**, 195410 (2005).